Pre-class Warm-up!!!

Let f : R^3 -> R be a function.
Select the best answer to complete the sentence.

The gradient of f is

a. a function R^3 -> R

b. a function R -> R^3

c, a function R^3 -> R^3 ✓

d, not defined.

e, none of the above.

$$\operatorname{grad} f = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

Question 2: What really important thing is different about today's Pre-class Warm-up!!! ?

4.3 Vector fields

We learn:

- What is a vector field
- Examples:
  flow of a fluid
  force fields
  gradient vector fields
- Flow lines

Types of question:

- sketch and recognize vector fields
- Verify that a given path is a flow line for some vector field
- Find a function with a specified vector field as gradient (qn 21, but not done in the text of the book)

Things we don't do (right now):

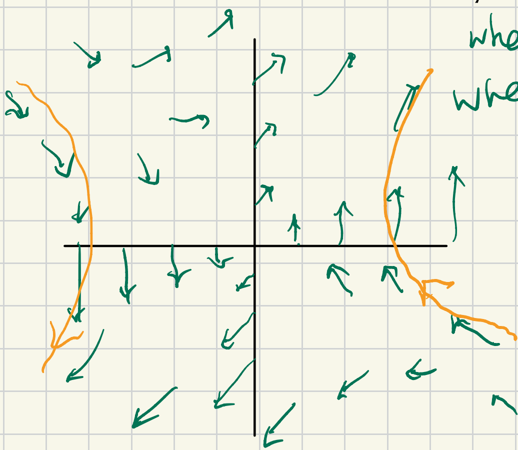
- Escape velocity
- Newton's gravitational law
- Coulomb's law
- Show that a vector field is not a gradient vector field (example 7)

Definition: a vector field on $\mathbb{R}^n$ is a function $\mathbb{R}^n \to \mathbb{R}^n$

Sketch the vector field $F(x,y) = (y, x+y)$

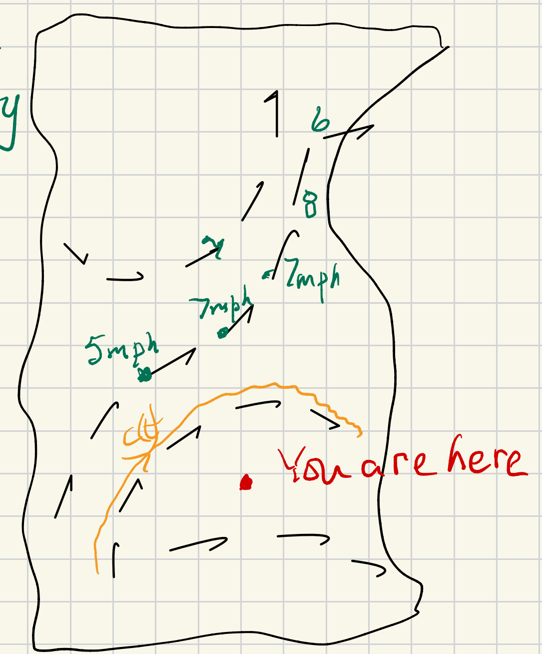
when $y=0$, $F(x,0)=(0,x)$

when $x=0$, $F(0,y)=(y,y)$



Definition: a flow line for a vector field F is a path $c:\mathbb{R} \to \mathbb{R}^n$ so that $c'(t)=F(c(t))$

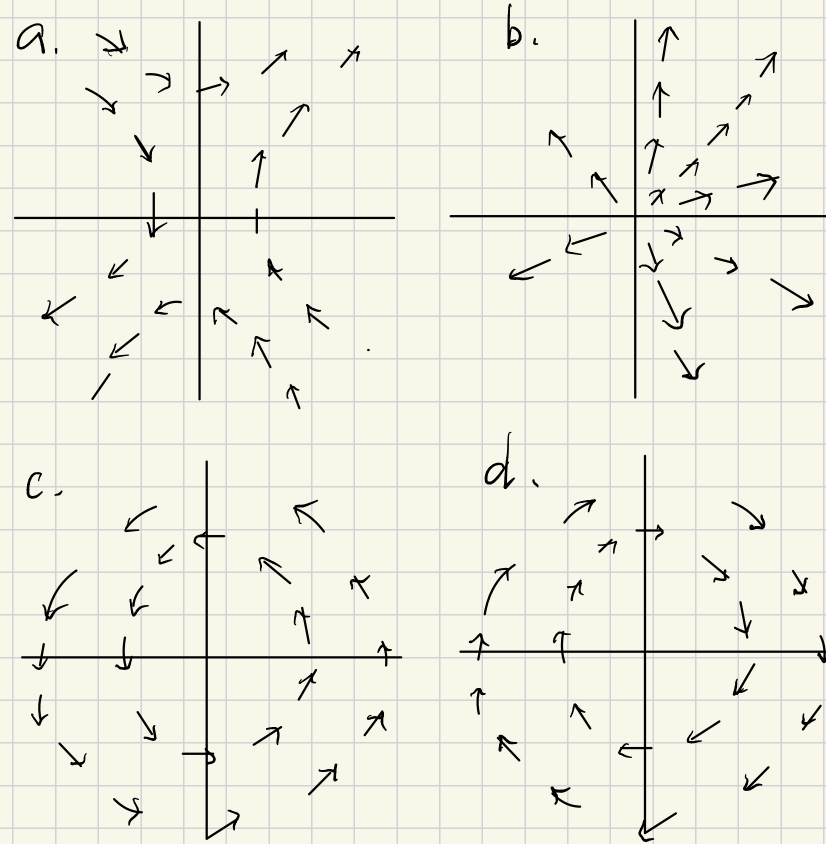
Example
Wind velocity gives a vector field on $\mathbb{R}^2$.



Match the fields to the pictures

1. $F(x,y) = (y, -x)$ d
2. $F(x,y) = (-y,x)$ c
3. $F(x,y) = (x,y)$ b
4. $F(x,y) = (y,x)$ a

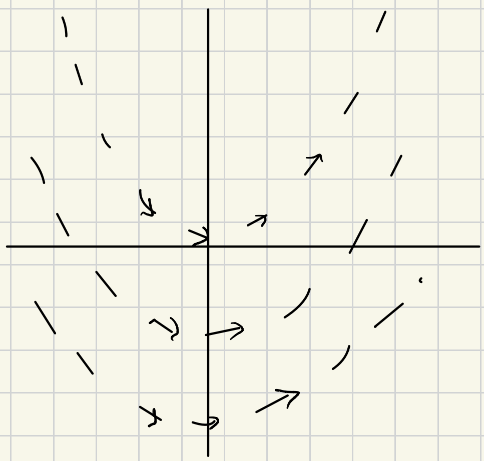
For 1. check when $y=0$:
$F(x,0) = (0,-x)$. On positive x-axis vectors point down



Like questions 15 - 20:
Show that c(t) = (t, t^2 / 2) is a flow line for the vector field F(x,y) = (1,x).

Solution. We check $c'(t) = F(c(t))$

$c'(t) = (1, t) = F(c(t)) = (1, t)$ ✓



Like question 21. Find a function f so that F(x,y,z) = (y^2, 2xy, 1) is the gradient of f (or show that such f does not exist).

Solution: We find $f(x,y,z)$ so that $\frac{\partial f}{\partial x} = y^2$, $\frac{\partial f}{\partial y} = 2xy$, $\frac{\partial f}{\partial z} = 1$.

Look at first equation: $f = xy^2 + a(y,z)$ for some a.

2nd equation $f = xy^2 + b(x,z)$

3rd eqn $f = z + c(x,y)$.

$f = xy^2 + z$ works! $\square$

4.4 Curl and divergence

We learn:

- The definitions of
  div F when F : R^n -> R^n
  curl F when F : R^3 -> R^3
- Notation $\nabla \cdot F = \operatorname{div} F$, $\nabla \times F = \operatorname{curl} F$
- Physical interpretations $\operatorname{grad} f = \nabla f$
- curl (grad f) = 0 and div (curl F) = 0
- the Laplacian. $\nabla \cdot (\nabla f) = \nabla^2 f$

What you don't need to memorize:

- the other formulas on page 255.

Types of questions:

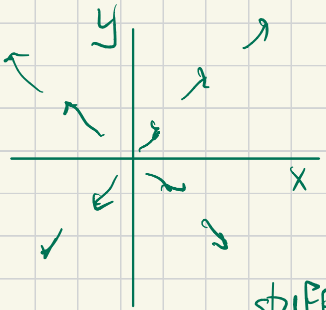
- calculate div and curl.
- Which composites make sense?
- Verify e.g. curl (grad f) = 0
- Scalar curl.

Definition Let F = (F_1, ... , F_n). The divergence of F is

$$\nabla \cdot F = \frac{\partial F_1}{\partial x_1} + \frac{\partial F_2}{\partial x_2} + \frac{\partial F_3}{\partial x_3} + \cdots + \frac{\partial F_n}{\partial x_n}$$

$$\nabla \cdot F : \mathbb{R}^n \to \mathbb{R}$$

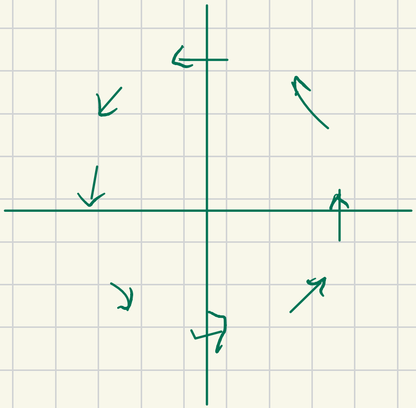
Examples: F(x,y,0) = (x,y,0) ; G(x,y,0) = (-y,x,0)



$$\nabla \cdot F = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial 0}{\partial z} = 2$$

everywhere.

It measures how much stuff is produced at each point



$$\nabla \cdot G = \frac{\partial(-y)}{\partial x} + \frac{\partial x}{\partial y} + \frac{\partial 0}{\partial z} = 0$$

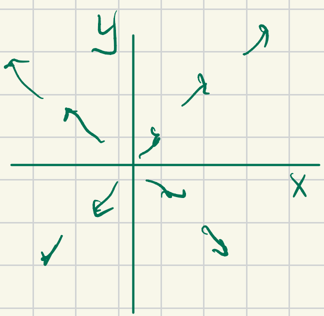
Definition Let F = (F_1, F_2 , F_3). The curl of F is

$$\operatorname{curl} F = \nabla \times F = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right)$$

$$= \det \begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{bmatrix}$$

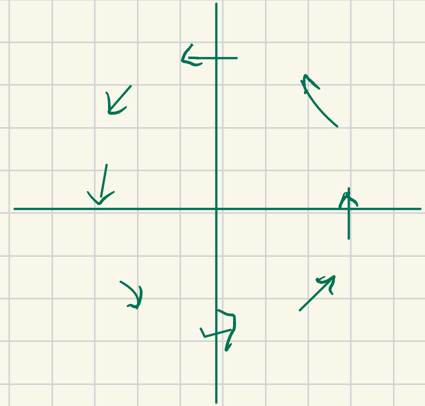
where $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$

Examples: F(x,y,0) = (x,y,0) ; G(x,y,0) = (-y,x,0)



$$\nabla \times F = \left(\frac{\partial 0}{\partial y} - \frac{\partial y}{\partial z}, \quad , \quad \right)$$

$$= (0, 0, 0)$$



$$\nabla \times G = \left(\frac{\partial 0}{\partial y} - \frac{\partial x}{\partial z}, \frac{\partial(-y)}{\partial z} - \frac{\partial 0}{\partial x}, \frac{\partial x}{\partial x} - \frac{\partial(-y)}{\partial y} \right)$$

$$= (0, 0, 2)$$

curl measures counterclockwise rotation about a vector, and points in the direction of that vector,